

# Formation of brine channels in sea-ice as habitat for micro-algae

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## Abstract

Brine entrapment between growing ice platelets in sea ice is an important habitat for a variety of  $CO_2$  - binding microalgae and therefore crucial in polar ecosystems. We microscopically describe the structure formation of ice platelets and develop a phase field model for pattern formation during solidification of the two-dimensional interstitial liquid by two coupled order parameters, the tetrahedrality as structure of ice and the salinity. These parameters describing the velocity of the freezing process and the velocity of structure formation, determine the phase diagram, the super-cooling and super-heating region, and the specific heat respectively. We use the model to calculate the short-time frozen microstructures and compare the morphological structure with the vertical brine pore space obtained from Xray computed tomography.

**Keywords:** brine channel distribution, sea-ice, freezing point suppression, phase field, pattern formation

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Ice-bound carbon consumption due to the organisms in the brine channels amounts to about 18% of the entire carbon consumption in the southern ocean. Therefore it is desirable to understand the formation of brine channels as one habitat for carbon-binding algae. Quantitative models [1, 2, 3] have investigated the brine channel volume, salinity profile or heat expansion, but unfortunately without pattern formation. The freezing process of salt water is one example of the solidification of binary alloys [4, 5] and first-order phase transition [6] within models of ice polluted with any salt as 'liquid jelly' [7]. A two-phase region consisting of pure ice crystals and water is known in the context of binary alloys as mushy layer [8, 9]. Percolation transitions are used in brine trapping during the solidification of seawater [10, 11, 12]. Highest cell abundances occur in this region, due to the higher porosity and due to the constant flushing with nutrient-rich seawater [13, 14]. Morphological stability theory is applied to the solidification of salt water [15].

Images of sea ice single crystals with X-ray computed tomography [16] show arrays of near-parallel intra-crystalline brine layers whose connectivity and complex morphology varies with temperature. The pore space turns out to be much more complicated than suggested by simple models of parallel ice lamellae and parallel brine sheets [17]. It is important to clarify the view taken in this paper from the beginning. Granular sea ice texture is sometimes thought to arise from the deposition of frazil ice crystals that form within the turbulent ocean interior and then rise buoyantly

to the ocean surface [18, 19]. In these settings the size of the settling crystals play a dominant role in controlling the structures observed. We consider here hypothetically the opposite view that these structures result from a thermodynamic instability during growth.

In order to describe realistic pattern formation and the phase transition on the same theoretical basis we use a phase field model for the solidification of the two - dimensional interstitial liquid. We will calculate the frozen micro-structures and will compare with the vertical brine pore space obtained from X-ray computed tomography [16, 20]. The aim is to present a model with a smallest possible number of microscopic parameters to be simulated or extracted from experiments. We find that three parameters are sufficient, the freezing, the structure, and the diffusivity parameter. Only the first two ones determine the phase diagram while the diffusivity enters the brine channel size. The linear stability analysis leads us then to the parameter range where structure can appear and the numerical solution will allow us to compare with the experimental data.

To discriminate between ice- and water molecules via a two-state function we use the "tetrahedrality" [21]

$$u = 1 - \frac{1}{15 \langle l^2 \rangle} \sum_{i,j} (l_i - l_j)^2, \quad (1)$$

as a measure of the state of order where  $l_i$  are the lengths of the six edges of the tetrahedron formed by the four nearest neighbors of the considered water molecule. For an ideal tetrahedron one has  $u = 1$  and the random structure is represented by  $u = 0$ . We assume the standard expansion

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of the energy function in powers of this order parameter [6, 22]

$$\frac{D_{\text{ice}}}{2}(\nabla u)^2 + \frac{a_1}{2}u^2 - \frac{a_2}{3}u^3 + \frac{a_3}{4}u^4 + \frac{h}{2}u^2v + \frac{D_{\text{salt}}}{2}v^2. \quad (2)$$

Here we have coupled additionally a second order parameter, the salinity  $v$ , in a specific manner which enables the conservation of the total mass of the salt. We demand a balance equation of the form  $\partial v / \partial t = -\nabla \cdot \vec{j}$  where the current is assumed to be proportional to a generalized force  $\vec{j} \sim \vec{F}$  which should be given in terms of a potential  $\vec{F} = -\nabla P$ . This potential in turn is expressed by the variation of the free energy density  $P = \delta f / \delta v$ . This procedure is nothing but the second law of Fick and we obtain an equation of the Cahn-Hilliard-type without the fourth derivation for the evolution of the salinity  $v$ .

The parameter  $a_1$  is the freezing parameter determining the phase transition, the structure parameter  $a_3$  is responsible for nonlinear behavior and  $D_{\text{ice}}$  and  $D_{\text{salt}}$  are the diffusion coefficients of ice and salt. The coefficient  $a_2$  is connected with an uneven exponent and is therefore responsible for the phase transition of first kind. The term  $h$  couples the ice and water and can be considered as reaction rate between water and ice. All these parameters depend on the temperature and can be scaled to only three relevant parameters among which the phase diagram is determined only by two, the dimensionless structure and freezing parameter as we will see.

Defining the reduced time  $\tau = D_{\text{salt}} a_2^2 t / h^2$ , the spatial coordinates  $\xi = a_2 x / h$ , the dimensionless order parameters of ice/water structure  $\psi = h^2 u / D_{\text{salt}} a_2$ , and the salinity  $\rho = h^3 v / D_{\text{salt}} a_2^2$ , we obtain the coupled order-parameter equations

$$\begin{aligned} \frac{\partial \psi}{\partial \tau} &= -\alpha'_1 \psi + \psi^2 - \alpha_3 \psi^3 - \psi \rho + D \frac{\partial^2 \psi}{\partial \xi^2} \\ \frac{\partial \rho}{\partial \tau} &= \frac{1}{2} \frac{\partial^2 \psi^2}{\partial \xi^2} + \frac{\partial^2 \rho}{\partial \xi^2}. \end{aligned} \quad (3)$$

This time-dependent Ginzburg-Landau differential equations couple the dynamics of the dimensionless order parameter  $\psi$  and the dimensionless salinity  $\rho$  depending only on three parameters, the freezing parameter  $\alpha'_1 = \frac{a_1 h^2}{a_2^2 D_{\text{salt}}}$ , the structure parameter  $\alpha_3 = \frac{a_3 D_{\text{salt}}}{h^2}$ , and the diffusivity  $D = \frac{D_{\text{ice}}}{D_{\text{salt}}}$  with  $\alpha'_1, \alpha_3, D > 0$ . The parameters  $\alpha'_1$  and  $\alpha_3$  describe the regions of ordered and non-ordered phase. This can be seen from the uniform stationary free energy density obtained from (3)

$$f(\Psi_0, \rho_0) = \frac{\alpha'_1}{2} \psi_0^2 - \frac{1}{3} \psi_0^3 + \frac{\alpha_3}{4} \psi_0^4 \quad (4)$$

where the temperature-dependent compound parameters  $\alpha_1(T) = \alpha'_1(T) + \rho_0$  and  $\alpha_0 = \frac{1}{2} \rho_0^2 - \gamma \rho_0$  appear which depend on the salinity  $\rho_0$ . Freezing-point depression occurs since  $\alpha'_1 + \rho_0$  corresponds to a higher temperature than  $\alpha'_1$ .

The temperature and salinity dependence of  $\alpha_3$  is supposed to be weak near the phase transition. At the lower

limit of the super-cooling region of fresh water [23, 24],  $T_c^0 = 233.15 K$ , the parameters  $\alpha'_1$  vanishes linearly for first-order phase transitions [22] such that we can assume  $\alpha'_1(T) = \tilde{\alpha}_1(T - T_c^0)$ . The freezing point depression in the framework of Landau-Ginzburg theory can be expressed therefore as

$$\Delta T = -\frac{\rho_0}{\tilde{\alpha}_1} = -\frac{D_{\text{salt}}}{\tilde{\alpha}_1} \frac{a_2^2}{h^2} \rho_0. \quad (5)$$

Introducing the salinity-dependent super-cooling temperature  $T_{c,s}^0 = T_c^0 - |\Delta T|$  the freezing parameters  $\alpha_1$  depends on the temperature according to

$$\alpha_1(T) = \rho_0 \frac{T - T_{c,s}^0}{|\Delta T|}. \quad (6)$$

Using the latent heat of the phase transition from water to ice  $\Delta H = 6 kJ/mol$  and a dissociation ratio of  $x = (n_{Na^+} + n_{Cl^-}) / n_{H_2O} = 1/50$ , the Clausius-Clapeyron relation yields a freezing point depression of  $\Delta T_{cc} = -\frac{xRT^2}{\Delta H} = -2K$  in agreement with the natural value of  $\Delta T = -1.9 K$ .

The free energy density has a minimum at  $\psi_0^0 = 0$  and a minimum/maximum for  $\psi_0^\pm = \frac{1}{2\alpha_3} (1 \pm \sqrt{1 - 4\alpha_1\alpha_3})$ . For  $\alpha_1 > 1/4\alpha_3$ , the minimum at  $\psi_0^0 = 0$  is the only allowed physical solution, which is the disordered state. As long as

$$\alpha_1 \leq \frac{1}{4\alpha_3} = \alpha_1(T_1) \quad (7)$$

a second relative minimum appears at  $\psi_0^+$ . Which of these minimis establishes the state of lowest free energy is found from the coexistence curve where these two local minimis are equal and  $f(\Psi_0^+) = f(\Psi_0^0) = 0$  which yields

$$\alpha_1(T_c) = \frac{2}{9\alpha_3}. \quad (8)$$

The coexistence curve is plotted as solid line in Fig. 1. Above the critical parameters  $\alpha_1(T_c) < \alpha_1(T) < \frac{1}{4\alpha_3}$  the ordered phase  $\psi_0^+ > 0$  is metastable whereas the non-ordered phase ( $\psi_0 = 0$ ) is stable. For small  $\alpha_1 \leq \alpha_1(T_c)$  the second minimum at  $\psi_0^+ > 0$  becomes deeper and the ordered phase  $\psi_0^+$  is the stable one. Therefore the absolute minimum changes discontinuously from  $\psi_0 = 0$  to  $\psi_0^+ > 0$  as plotted in Fig. 1(b). The jump at  $T_c$  is a measure for the latent heat during the first order phase transition between water and ice.

We identify now the upper borderline of stable structure formation (8) with the super-cooling temperature since this is the line where structure, i.e. ice formation is possible at all. In the same manner the borderline of metastable structure (7) represents the super-heating temperature. The shaded area in Fig. 1 describes the super-cooling region between  $T_c$  and  $T_c^0$ . Above this area we find the super-heating region for  $T_c < T < T_1$ . From (7) and (8) a relation between the super-cooling temperature  $T_c^0$ , the

freezing-point temperature and the super-heating temperature  $T_1$  reads

$$T_1 = \frac{9}{8}T_c - \frac{1}{8}T_c^0. \quad (9)$$

After a super-heating of more than  $5^\circ\text{C}$ , homogeneous nucleation occurs in the metastable state [25]. For fresh water ( $T_c = T_0 = 273.15\text{K}$  and  $T_{c,s}^0 = T_c^0 = 233.15\text{K}$ ) from equation (9) follows that  $T_1 = 278.11\text{K}$  ( $4.96^\circ\text{C}$ ) as the upper limit of super-heating in agreement with the experiment [25].

The linear stability analysis for the two local minim around the disordered phase  $\psi_0^0$  and the ordered phase  $\psi_0^+$  with  $\bar{\rho} = \bar{\rho}_0 \exp[\lambda(\kappa)\tau + i\kappa\xi]$  leads to the two possible growth rates

$$\lambda_{1,2} = -[(D+1)\kappa^2 - \aleph \pm \sqrt{\Delta}]/2 \quad (10)$$

with  $\Delta = [(D-1)\kappa^2 - \aleph]^2 + 4\kappa^2\psi_0^2 > 0$  and  $\aleph = -\alpha_1 + 2\Psi_0 - 3\alpha_3\Psi_0^2$  which takes the value  $\aleph = -\alpha_1$  for the fixed point  $\Psi_0^0 = 0$  and  $\aleph = \psi_0 - 2\alpha_3\psi_0^2$  for  $\Psi_0^\pm$ . Time-oscillating structures would appear only if  $\text{Im}\lambda(\kappa) \neq 0$ , i.e.  $\Delta < 0$ , which is never the case in our model.

An unstable fixed point  $\lambda(\kappa) > 0$  allows any fluctuation with a wave-vector  $\kappa$  to grow exponentially in time. For the fixed point representing the disordered phase,  $\psi_0 = 0$  and  $\rho_0 = \text{const}$ ,

$$\lambda_{1,2} = \frac{1}{2} [-(D+1)\kappa^2 - \alpha_1 \pm |(D-1)\kappa^2 + \alpha_1|] < 0 \quad (11)$$

and no structure formation occurs in this state which was expected for the disordered phase, of course.

We can only have positive  $\lambda(\kappa)$  if the values of  $\kappa$  are restricted to the region between the zeros of  $\lambda(\kappa)$ , which is  $\kappa^2 \in (0, \psi_0^+[1 - (2\alpha_3 - 1)\psi_0^+]/D)$ . Discussing separately the cases  $\alpha_3 >, < 1/2$  and recombining results we obtain the range for possible structure formation

$$\begin{aligned} 2 > \alpha_3 > 1 : \quad & \frac{1}{4\alpha_3} \left( 1 - \frac{1}{(2\alpha_3 - 1)^2} \right) < \alpha_1 < \frac{2}{9\alpha_3} \\ 1 > \alpha_3 > 0 : \quad & 0 < \alpha_1 < \frac{2}{9\alpha_3} \end{aligned} \quad (12)$$

represented in Fig. 2 as a phase diagram for the freezing and structure parameters.

The structure parameters  $\alpha_1$  determines the brine channel formation. A small  $\alpha_1$  means low temperatures or low salinities and consequently a freezing process. with a uniform ice phase for sufficiently large  $\alpha_3$  and a precipitate of salt. In contrast at higher  $\alpha_1$  there are higher temperatures or higher salinities inducing a melting with a uniform liquid water phase and dissolved salt. The spatial structures can only appear in the instability region which starts at the maximal point  $\alpha_1 = 1/9$  at  $\alpha_3 = 2$ . The description of the instability region does not involve a restriction on the diffusivities of salt and water. This is different from the model of [26], which describes structure formation in sea-ice in terms of Turing structures.

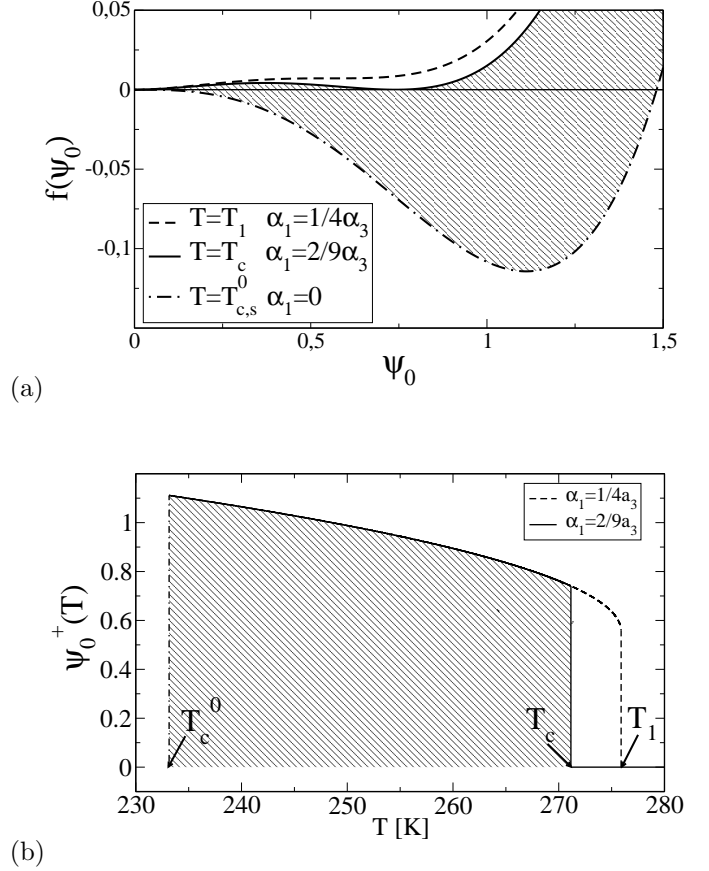


Figure 1: Condition for a first-order phase transition. (a) The free energy density  $f$  versus the uniform dimensionless order parameters (tetrahedrality) for some freezing parameters  $\alpha_1$  and the structure parameters  $\alpha_3 = 0.9$  representing the super-cooling region for freezing of hexagonal ice  $T_{c,s}^0 < T_c$  (shaded area) and the super-heating region  $T_c < T < T_1$ . (b) Dependence of the absolute minimum of the free energy density on  $\alpha_1$  (solid line) in temperature-dependent representation. The dashed line corresponds to the position of the second minimum.

Before solving (3) numerically we use (6) to determine the values of  $\alpha_1$  and  $\alpha_3$  in terms of water properties. The structure parameter  $\alpha_3 = 0.9$  leads to a freezing point temperature of  $-1.9^\circ\text{C}$  ( $T_c = 271.25\text{K}$ ) for seawater of salinity  $35\text{g/kg}$  ( $\rho_0 = 0.6\text{mol NaCl}/53\text{mol H}_2\text{O} = 0.0113$ ). The size of solidification structures depends on the super-cooling relative to the freezing point  $T_c$ . The higher the super-cooling, the more rapidly water freezes and the smaller the structures become. Our choice of the freezing parameter  $\alpha_1 = 0.2$  represents a temperature  $T_2 = -8.2^\circ\text{C}$ . A structure parameter  $\alpha_3 = 0.9$  represents therefore a realistic description of super-cooling pure water. The specific heat  $c$  is dependent on  $\alpha_3$  as

$$\begin{aligned} c &= -T \frac{\partial^2 f(\psi_0^+(T))}{\partial T^2} \Big|_{T=T_c^0} \\ &= \frac{\tilde{\alpha}_1^2 T_c^0}{2\alpha_3} \left( 1 + \frac{3}{\sqrt{1 + 36\tilde{\alpha}_1\alpha_3(T_c^0 - T)}} \right) \end{aligned}$$

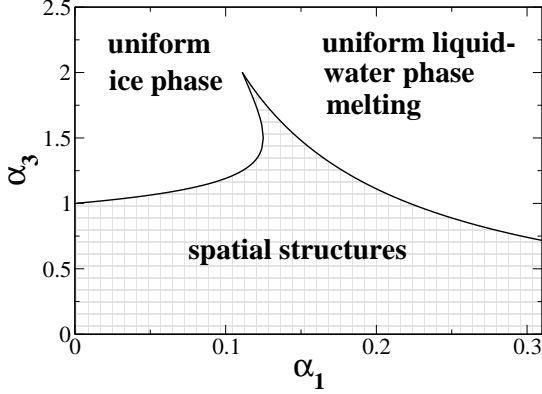


Figure 2: The instability regions of the fixed point  $\psi_0^+$  and  $\rho_0 = \text{const}$  as phase diagram together where spatial structures can occur (checked region).

$$= \frac{4}{81} \frac{T_c^0}{\alpha_3^3 (T_c - T_c^0)^2}. \quad (13)$$

We set the energy scale to be the difference of the latent heat of water freezing  $K_E = L(0^\circ\text{C}) - L(-40^\circ\text{C}) = 98\text{J/g}$  [23]. The resulting specific heat in our theory yields  $c_{\text{spec}} = K_E c = 2.14\text{J/gK}$  which compares well with the experimental value of  $c_{\text{exp}} = 2\text{J/gK}$ . This shows that the chosen structure parameter  $\alpha_3 = 0.9$  is in agreement with the specific latent heat too.

The parameters  $\alpha_1$  and  $\alpha_3$  define the local portion of the free energy in a system with uniform order parameter and salinity. The spatial inhomogeneity of the system is described by the third parameter of the model  $D = D_{\text{ice}}/D_{\text{salt}}$ . At the freezing point temperature of seawater of  $-1.9^\circ\text{C}$ , the study in [27] predicts  $D_{\text{salt}, -1.9^\circ\text{C}} = 0.62 \times 10^{-5}\text{cm}^2/\text{s}$ . The  $D_{\text{ice}}$  can be linked to the reorientation rate of the  $\text{H}_2\text{O}$ -molecules and the correlation length [28] which leads with realistic numbers [29, 30] to  $D_{\text{ice}} = 0.33 \times 10^{-5}\text{cm}^2/\text{s}$  and finally to a ratio  $D_{\text{ice}}/D_{\text{salt}} = 0.47$ .

Now we integrate the equation system (3) numerically in one and two space dimensions by the exponential time differencing scheme of second order (ETD2) [31] for a stiff differential equation of the type  $\dot{y} = ry + z(y, t)$  with a linear term  $ry$  and a nonlinear part  $z(y, t)$ . The linear equation is solved analytically and the integral over the nonlinear part is approximated by a proper finite differencing scheme.

The evolution of the order parameter  $\psi$  and the salinity  $\rho$  in one and two dimensions is shown in Fig. 3. The quantities  $\psi$  and  $\rho$  are complementary in phase. Due to the second equation of (3) the conservation of salinity  $\int dx \rho(t, x) = c$  is ensured. We can absorb this mean salinity into  $\rho \rightarrow \rho - c$  leading to a mere shift in  $\alpha_1' - c + \rho_0 = \alpha_1$  which means we consider with  $\rho$  the deviations from a mean salinity  $c$  and the total salinity remains positive. Regions of high salinity correspond to the water phase and regions of low salinity to ice domains. We see that one sin-

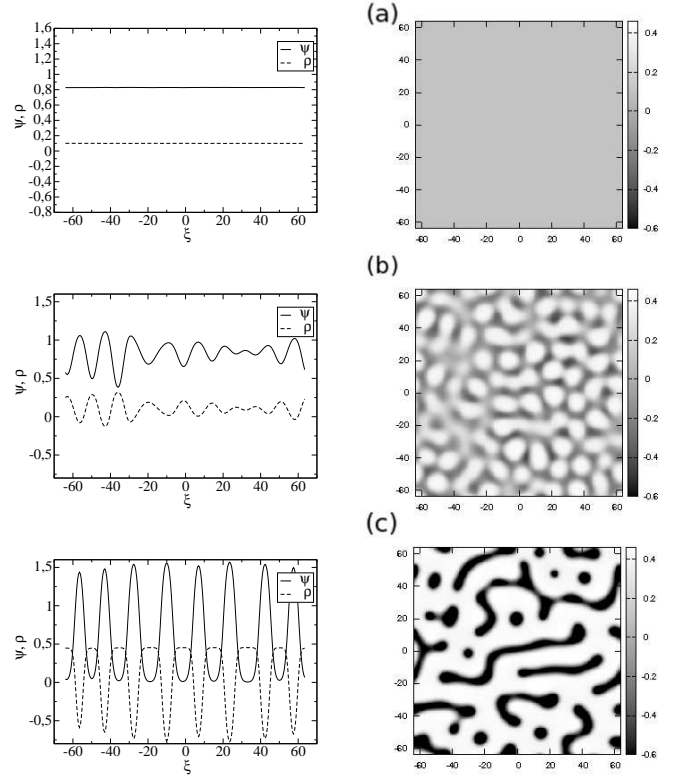


Figure 3: Time evolution of the order parameter  $\psi$  and salinity  $\rho$  as deviation from mean salinity for 1D (left) and the salinity for 2D (right) versus spatial coordinates for  $\tau = 10, 150, 500$  (from above to below) with the initial random distribution  $\psi(\tau = 0) = 0.9$  and  $\rho(\tau = 0) = 0.1 \pm 0.001N(0, 1)$ . The parameters are  $\alpha_3 = 0.9$ ,  $\alpha_1 = 0.1$ , and  $D = 0.5$ .

gle mode develops given by the wave number  $\kappa_c$ . Similar to the one-dimensional case, we see the formation of one dominant wavelength also in two dimensions. this can be understood as the maximum of unstable wavelengths (10) which becomes

$$\kappa_c^2 = \frac{\psi_0}{(D-1)^2} \left[ (D-1)(1-2\alpha_3\psi_0) - 2\psi_0 \right] + \frac{(D+1)\psi_0^{1/2} \sqrt{(D-1)(2\alpha_3\psi_0-1) + \psi_0}}{\sqrt{D}} \quad (14)$$

The critical wave number sets the length scale on which phase separation occurs and is visible as the dominating coarse graining mode in figure 3.

To compare with the experiments we suggest three types of comparisons: (i) morphology, (ii) percolation threshold and (iii) structure size where our model describes realistic parameters. We will start with the morphology. For the web of brine channels one observes different textures for instance granular ice, columnar-granular structures or plate ice. Fig. 4(b) shows a measurement yielding granular texture [20] without prevalent orientation. In figure 4(c) we have chosen the best fit of the former Turing-model [26] to the structure. If we compare the simulation according to the here presented phase field model in Fig.

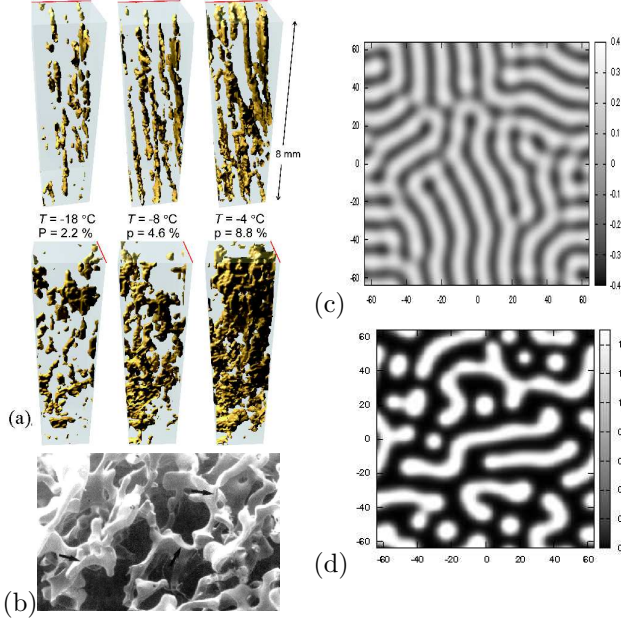
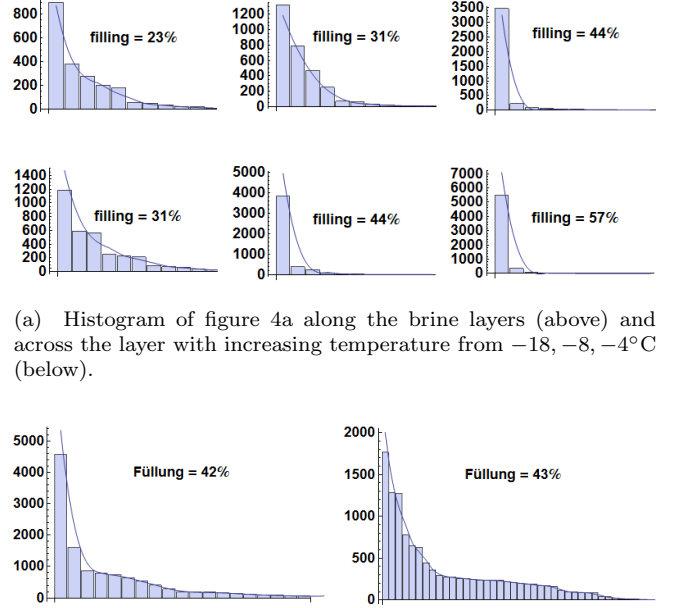


Figure 4: (a) Imaging brine pore space with X-ray computed tomography (image from [16]). The upper images shows the view approximately along the brine layers. The view across the brine layers is shown in the bottom images. (b) Scanning electron microscopy image of a cast of brine channels [20], (c) Turing structure after long time [26], (d) long-time phase field structure from figure 3.

4(d) the texture of the cast of brine channels is better described by the phase-field model than by the Turing image. Please note that the three parameters of the Turing model had been adjusted to fit the structure as best as possible. Here with the phase-field model we have chosen parameters according to the thermodynamic properties of water and have obtained the structure as a consequence of these parameters.

The structure of the brine pore space of sea ice single crystals [16] is seen in iso-surface plots in Fig. 4(a). The upper images clearly show near-parallel intra-crystalline brine layers. The view across the layers (bottom images) show brine layer textures much more complicated than suggested by the simple model of parallel ice lamellae and parallel brine sheets illustrated in Fig. 4(b). Depending on the temperature, the images show a brine pore porosity from  $p = 2.2\%$ – $8.8\%$ . The connectivity increases with porosity as the pore space changes from isolated brine inclusions at  $p = 2.2\%$  to extended, near-parallel layers at  $p = 8.8\%$ . PRINGLE *et al.* [32] characterized the thermal evolution of the brine pore space with percolation theory and demonstrated a connectivity threshold at a critical volume fraction  $p_c = 4.6\%$ . Below  $p_c$  there are no percolating pathways spanning a sample, i.e. the brine is trapped within the intra-crystalline brine layers. Lets quantify this statement by a cluster-size analysis of the figures 4a where the corresponding histograms are given in figure 5. As one sees the percolation transition is visible around 44% filling around  $-8^\circ\text{C}$ – $-4^\circ\text{C}$ . We compare with our simulation with two parameter  $\alpha_3 = 0.9$  and  $\alpha_3 = 1$  we see that we are



(b) Histogram of the numerical result in figure 3 with  $\alpha_1' = 0.1$ ,  $D = 0.5$ , and  $\alpha_3 = 0.9$  (left) compared to  $\alpha_3 = 1$  (right).

Figure 5: The histograms of connected clusters.

just at the percolation threshold with our simulation with comparable histograms. This shows that the used experimental parameter for our model allows to describe realistic morphological structures.

Next we compare the size of the obtained structure first with the size of pure sea ice platelets [15, 16, 12], separating regions of concentrated seawater. The fastest-growing wave-vector  $\kappa_c(D, \alpha_1, \alpha_3)$  sets the length scale on which phase separation occurs. The size of the structure can be estimated by  $2\pi/\kappa_c$ . With the help of (5) and remembering the dimensionless values introduced before (3), the critical domain size of the phase field structure as a function of the freezing point depression takes the value

$$\lambda_c = \frac{2\pi}{\kappa_c} = \frac{2\pi}{\kappa_c} \frac{h}{a_2} = \frac{2\pi}{\kappa_c} \sqrt{\frac{D_{salt}\rho_0}{\tilde{a}_1|\Delta T|}} \quad (15)$$

where one gets with the parameters  $\alpha_3 = 0.9$ ,  $\alpha_1 = 0.2$  and  $D = D_{ice}/D_{salt} = 0.5$  a dimensionless pattern size of 13.81. Our choice of the freezing parameter  $\alpha_1 = 0.2$  represents a super-cooling  $\Delta T_{sup} = 6.3\text{K}$ . The rate of re-orientation of the  $\text{H}_2\text{O}$ -molecules determines  $\tilde{a}_1 = 1/[(T_0 - T_c^0)\tau_d(T_0)] = 1250\text{K}^{-1}\text{s}^{-1}$ . With these parameters we obtain from (15) a critical domain size  $\lambda_c = 0.8\mu\text{m}$  in agreement with the sea ice platelet spacing  $\lambda_{max} \approx 1\mu\text{m}$  obtained from morphological stability analysis [15] or percolation theory [16, 12].

Second, we consider the size of phase-field structures for natural conditions which is given by the upper limit of the instability region shown in Fig. 2. With a structure parameter  $\alpha_3 = 1.99$  and a freezing parameter  $\alpha_1 = 0.111482$  one has a realistic description of seawater at  $0.032\text{K}$  super-

cooling and a lower limit of the super-cooling region of fresh water at  $-18.78^{\circ}\text{C}$ . For this growth condition we obtain the dimensionless structure size  $2\pi/\kappa_c = 4975.25$  and using equation (15) the critical domain size is  $\lambda_c = 198\mu\text{m}$  in agreement with the observed values. Brine inclusions [20] have scales from  $3 - 1000\mu\text{m}$ , where the average dimensions is typically  $200\mu\text{m}$ .

To summarize, a model for brine channel formation in sea ice is used which consists of two coupled order parameters, the tetrahedrality and the salinity preserving the mass conservation of salinity. The linear stability analysis provides a phase diagram in terms of two parameters indicating the region where spatial structures can be formed due to the instability of the uniform ordered phase. The region of instabilities is determined exclusively by the freezing parameter and the specific heat or structure parameter and not by the diffusivity as it was the case in the Touring model. This allows to link these parameters to thermodynamical properties of water like super-heating, super-cooling, freezing temperature and latent specific heat simultaneously. With the help of these parameters we solve the time-dependent coupled evolution equations and find a brine channel texture in agreement with the experimental values. That the physical justification of the parameters by other properties of water leads here to a better description of the brine channel texture we attribute to the mass conservation invoked in the present model.

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